Abstract

This paper considers optimal synthesis of a special type of four-bar linkages. Combination of this optimal four-bar linkage with on of it's cognates and elimination of two redundant cognates will result in a Watt's six-bar mechanism, which generates straight and parallel motion. This mechanism can be utilized for legged machines. The advantage of this mechanism is that the leg remains straight during it's contact period and because of it's parallel motion, the legs can be as wide as desired to increase contact area and decrease the number of legs required to keep body's stability statically and dynamically.

“Genetic algorithm” optimization method is used to find optimal lengths. It is especially useful for problems like the coupler curve equation which are completely nonlinear or extremely difficult to solve.

1. Introduction

Legged machines have been used for at least a hundred years and are superior to wheels in some aspects:

- A US Army investigation [1] reports that about half the earth's surface is inaccessible to wheeled or tracked vehicles, whereas this terrain is mostly exploited by legged animals.
- Legged locomotion should be mechanically superior to wheeled or tracked locomotion over a variety of soil conditions [2] and certainly superior for crossing obstacles [3].

In spite of all these advantages, legged machines have not been used practically in industry as a replacing of wheeled machines. Some reasons are given below:

- The robot kinematics and dynamics are nonlinear, difficult to accurately model and simple models are generally not adequate [4]. Furthermore, the dynamics depend on which legs are on the ground, and might therefore be considered as switching. Robot parameters (centre of mass position, amount of payload etc) are not known exactly and might also vary.
- A legged system has a lot of degrees of freedom. In order to allow a complet

decoupled motion over irregular terrain, at least three degrees of freedom per leg are required. This results in 12 actuators for a four-legged robot. For this reason, they are very expensive and hard to control [3].

This paper describe a one DOF six-bar mechanism which in spite of it's simplicity, simulates walking of animals very well. The six-bar is the result of combination of two four-bars. Path generation could be very difficult because the coupler curve equation is completely nonlinear and extremely hard to solve. There have been some efforts to optimize four-bar linkages by different optimization algorithms for different purposes. For example Differential Evolution and Centroid of Precision Points (GCPP) is applied to the synthesis of four-bar linkages for path generation with prescribed timing, where the coupler point is required to pass through a number of precision points within a prescribed accuracy level and in the correct order, and for the generation of families of coupler curves[5]. Numerical methods are also used to optimize the mass distribution of links to reduce the change of joint forces. The mass, the center position of mass and the moment of inertia of the moving links are taken as the optimizing variables [6]. Elsewhere it has been used to reduce design space [7] or generating a user-specified curve using artificial neural networks (ANNs) optimization algorithm [8].

2. Kinematic synthesis of the linkage

2.1. Introducing the linkage

Fig. 1 shows three known straight line generating mechanisms. None of these three mechanisms satisfy Grashof crank-rocker inequality. In addition Roberts and Evan mechanisms are not suitable as a leg because of the crunodes in their path and complex coupler curves. And it can be proved that Chebyshev mechanism is the cognate of the linkage shown in Figure 2. This linkage is a special form of four-bar linkages, where the triangle of coupler is a line. The coupler curve is symmetric and can be almost straight in one part. This mechanism, in spite of Chebyshev four-bar is a fully crank-rocker type. However as is shown in Figure 2, the shape of coupler curve can be completely different with a little change in lengths of the links. The optimization process aims to find optimum lengths to approach a coupler curve with straight line part.

Evan or "grasshopper"

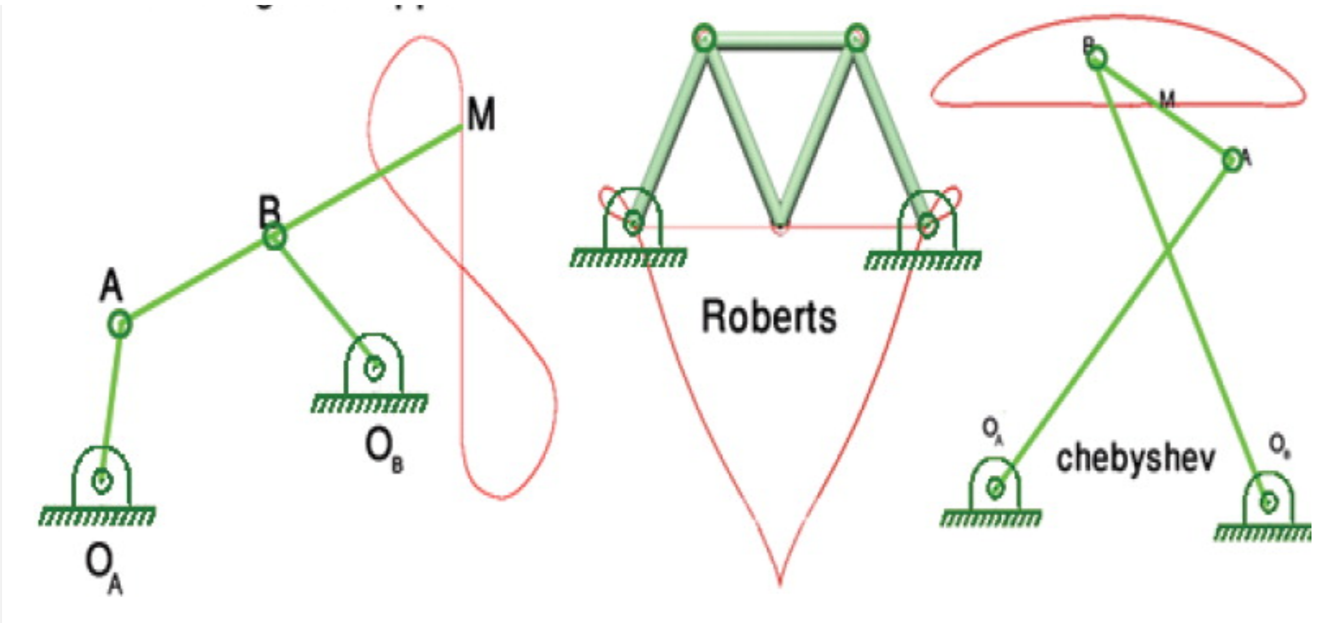


Figure 1. Three known straight line generating mechanisms

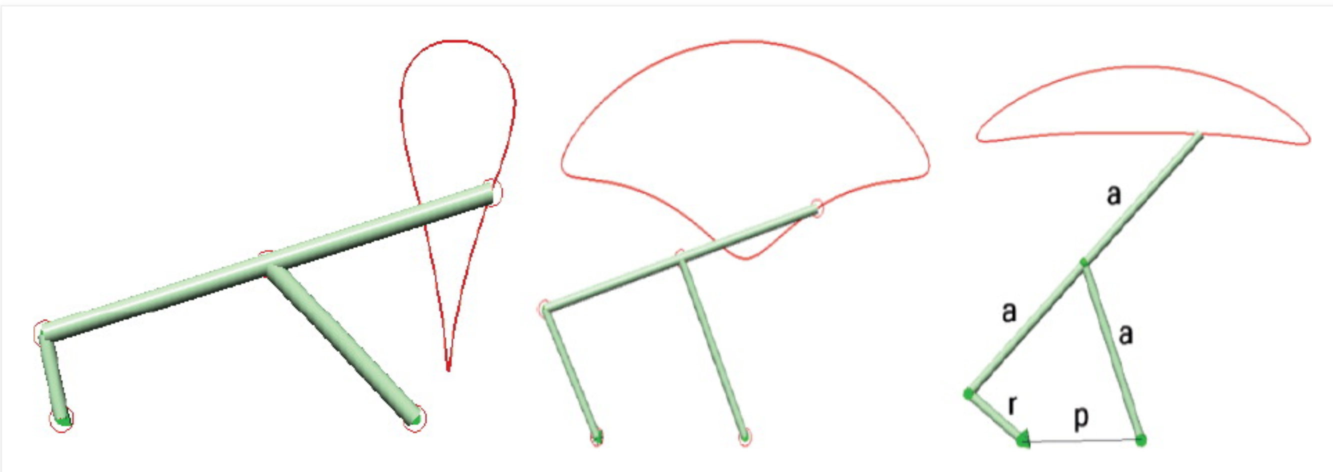
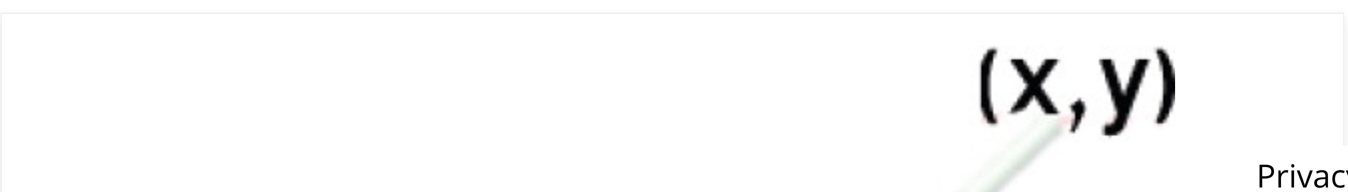


Figure 2. Different shapes of coupler curves generated by the same type of mechanism

2.2. Coupler curve formulization

The linkage is label as shown in Figure 3. The procedure contains eliminating two angels (α, γ) from coupler curve equation.



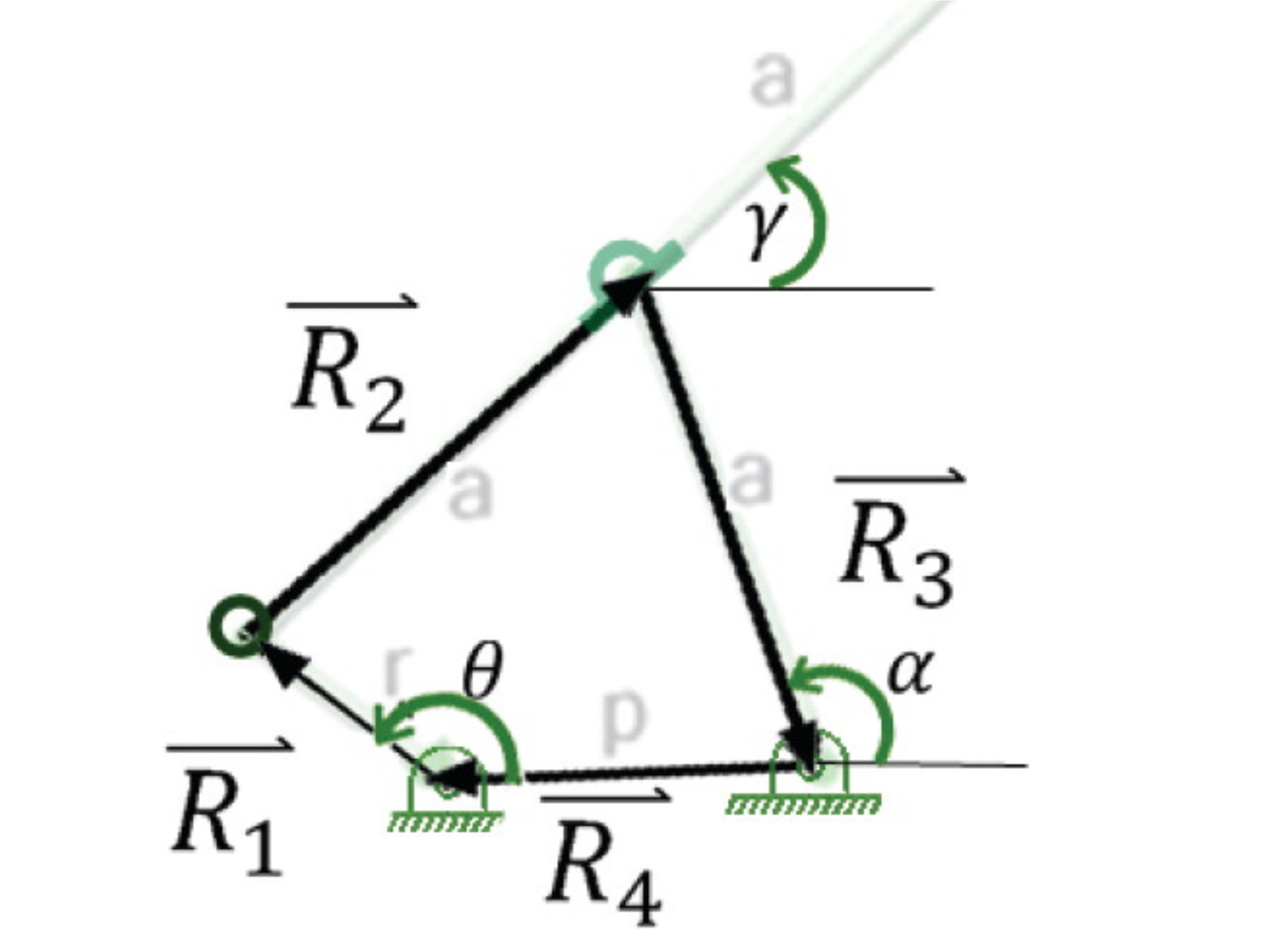


Figure 3. Linkage notation

$$\overrightarrow{R_1} + \overrightarrow{R_2} + \overrightarrow{R_3} + \overrightarrow{R_4} = 0 \tag{1}$$

$$asin\gamma + rsin\theta - asin\alpha = 0 \tag{2}$$

$$rcos\theta + acos\gamma - \text{acos } \alpha - p = 0 \tag{3}$$

By eliminating α, combining eq.2 with eq.3 and summarizing we have:

$$r^2 + p^2 + 2ar\ sin\theta\ sin\gamma + 2arcos\theta\ cos\gamma - 2r\ cos\theta - 2a\ cos\gamma = 0 \tag{4}$$

In the other hand the position of coupler point is:

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$$x = r\ cos\theta + 2a\ cos\gamma \tag{5}$$

$$y = r\ sin\theta + 2a\ sin\gamma \tag{6}$$

By finding γ versus θ from eq.4, and substituting in eq.5 and eq.6 we have:

$$x = p + \frac{r\ sin\theta}{\sqrt{(p^2+r^2-2\ p\ r\ cos\theta)}}\ \sqrt{4\ a^2-(p^2+r^2-2\ p\ r\ cos\theta)} \tag{7}$$

and

$$y = \frac{\left(p-r\ cos\theta\right)\sqrt{4\ a^2-(p^2+r^2-2\ p\ r\ cos\theta)}}{\sqrt{(p^2+r^2-2\ p\ r\ cos\theta)}} \tag{8}$$

or:

$$x = p + \frac{r\ sin\theta}{d}\ \sqrt{4a^2-d^2} \tag{9}$$

$$y = \frac{|p-r\ cos\theta|}{d}\ \sqrt{4a^2-d^2} \tag{10}$$

Where:

$$d = \sqrt{p^2+r^2-2\ p\ r\ cos\theta} \tag{11}$$

2.3. Penalty function

In the optimization process all equality and inequality constraints and conditions should be considered. The first inequality constraint comes from that there is no negative length:

$$a,\ r,\ p > 0 \tag{12}$$

$$4a^2-d^2 > 0 \tag{13}$$

or:

$$\sqrt{p^2+r^2-2\ p\ r\ cos\theta}\ -2\text{a} < 0$$

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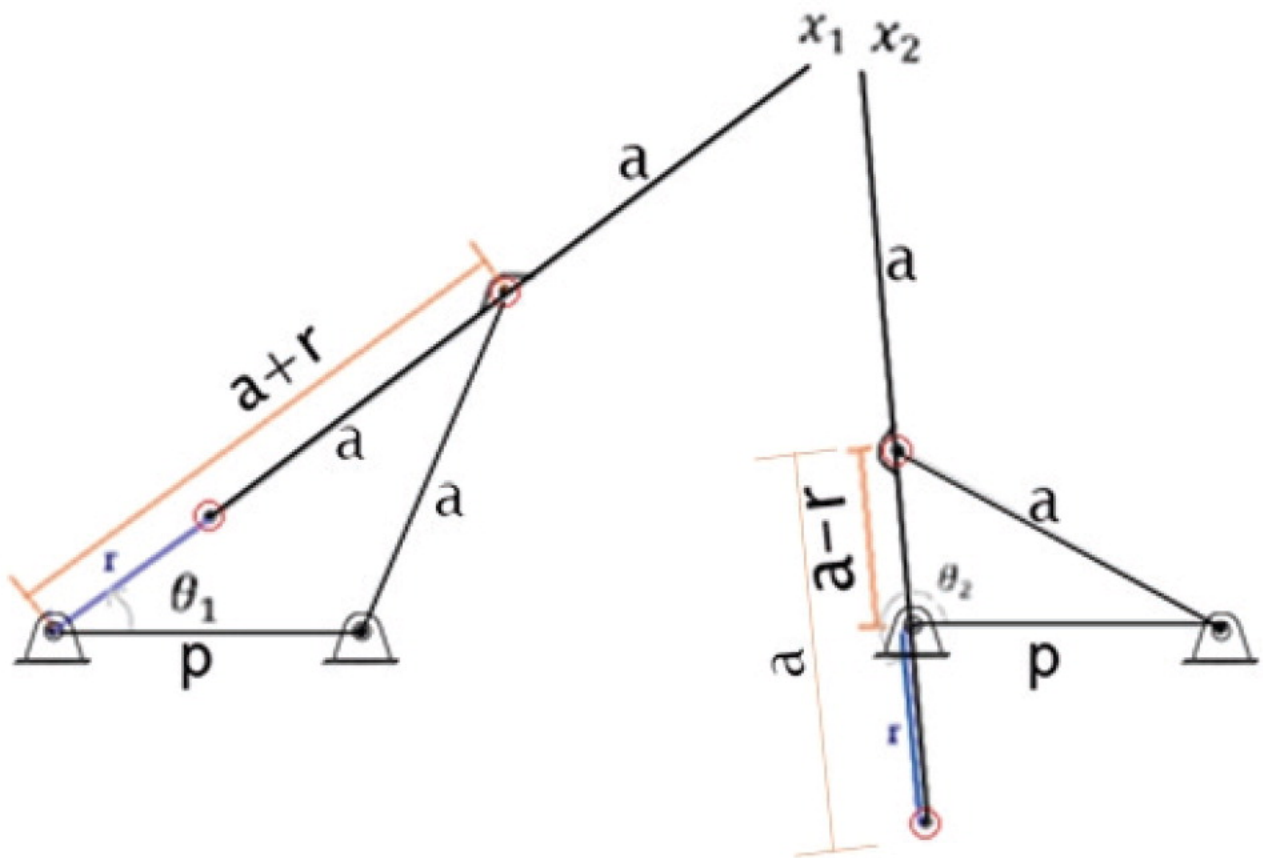

Figure 4. Critical points and angles

The critical position occurs when rocker and coupler link (i.e. r and a) are parallel. The value of x changes between these two positions. Considering the rotation of “r” counter clockwise, “x” will decrease constantly from x_1 to x_2 but “y” should remain constant (straight course).

$$x_1 = \frac{\left(\frac{1}{2}r+a\right)\left(r^2+2ar+p^2\right)}{p(a+r)} \tag{15}$$

$$x_2 = \frac{\left(-\frac{1}{2}r+a\right)\left(r^2+2ar+p^2\right)}{p(a-r)} \tag{16}$$

3. Optimization of an arbitrary initial four-bar

3.1. Introductions to optimization and genetic algorithm (GA)

There are two common requirements in kinematic synthesis of mechanisms: path generation and motion generation. In dimensional synthesis there are two approaches: synthesis of precision points and approximate or optimal synthesis.

Precision point synthesis implies that the point on the coupler plane passes through a certain number of desired (exact) points, but without the possibility of controlling a structural error on a path out of those points. Precision point synthesis is restricted by the number of points which must be equal to the number of independent parameters defined by the mechanism. The maximum number of points for a four-bar linkage is nine. If the number of equations generated by the number of exact points is smaller than the number of projected variables, then there is a selection of free variables, so that the problem of synthesis does not have a single-valued solution. When the number of precision points increases, the problem of precision point synthesis becomes very nonlinear and extremely difficult for solving, and the mechanism obtained by this type of synthesis is in most cases useless: because dimensions of the mechanism members are in disproportion, or the obtained solutions are in the form of complex numbers so there is no mechanism. The maximum number of precision points on the path of the coupler in a four-bar-linkage is nine in uncoordinated motion.

It is seen that the synthesis of mechanisms by methods of precision points” is restricted by the number of given points, and the increase of precision points to more than nine is practically impossible.

Optimal synthesis of mechanisms is, in fact, a repeated analysis for a random determined mechanism and finding of the best possible one so that it could meet technological requirements, and it is most often used in dimensional synthesis, which implies determination of elements of the given mechanism (lengths, angles, coordinates) necessary for creation of the mechanism in the direction of desired motion. The optimization algorithm contains the objective function defined by the problem of synthesis and it represents a set of mathematical relations; it must be chosen in such a way that the conditions perform desired tasks presented in a well defined mathematical form. In the optimization algorithm, the objective function is given a numerical value for every solution and it would be ideal for the objective function to have the result in a minimum, which corresponds to the best mechanism possible in a region near the primitive selected solution. Which, should perform a technological procedure, but it is difficult to be achieved because of very complex problems.

The objective function may contain various restrictions, such as:

Restriction of ratio of lengths of the members, prevention of negative lengths of the members, restrictions regarding transmission angles, satisfying Grashof crank-rocker inequality, etc.

To eliminate such results, a constraint function is also defined.

In this paper, genetic algorithm is used to optimize the path generated by a four-bar linkage which will result in optimization of the motion of a the legs connected to Watt's six-bar mechanism.

The aim is to find an optimized solution in the vicinity of the primitive solution. So we are looking for a regional extremum for the objective function instead of a global extremum.

The Genetic Algorithm (GA) is a simplistic numerical simulation of a natural population evolving over time. In the context of an optimization problem, each individual of the population represents a solution to the problem. The population is subject to operators that are similar to natural evolutionary processes. In particular, each solution has a fitness value, the solutions are combined together in a reproductive-like process and stronger solutions contribute more offspring [9].

3.2. Objective function formulation

The first step is to assign an ideal value to each point selected in the initial coupler curve which has the greatest deviation from straight line. The least square of the distance between ideal points selected and the parametric values of y=f (a, r, p) corresponding to that ideal points, will be the objective function that must be optimized (i.e. minimized).

$$objfun(a,\ r,\ p) = \sum_{i=1}^{i=n} \left(y(a,\ r,\ p,\ x_i) - y(a_0,\ r_0,\ p_0,\ x_i)\right)^2 \tag{17}$$

Table 1. Initial and optimal linkage dimensions

	a	r	p
Initial values	25	13	22
Optimized values	25.5	11.63	[illegible]

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Where objective function is in regional minimum (Objfun=0.00163)

The coupler curve is drawn by formula (figure 6.a) and also by kinematic analysis software (figure 6.b) which are almost the same.

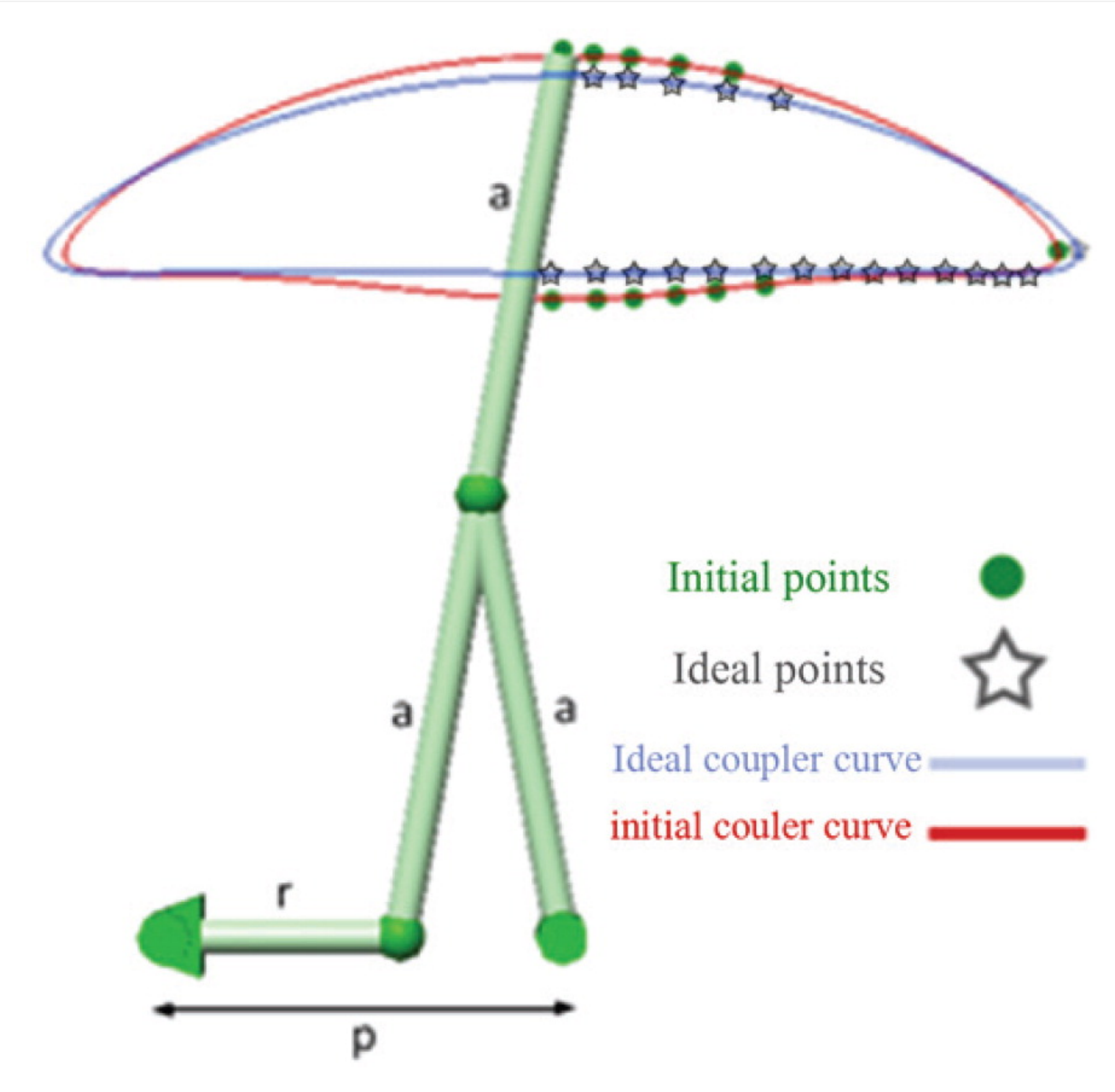

Figure 5. Initial versus Ideal path

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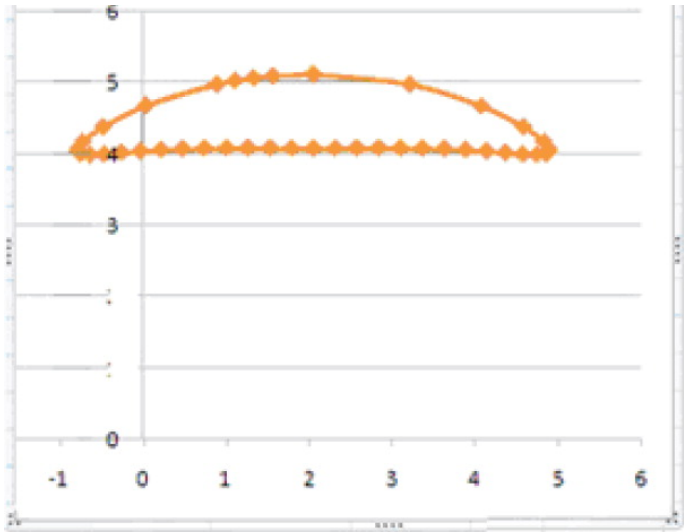
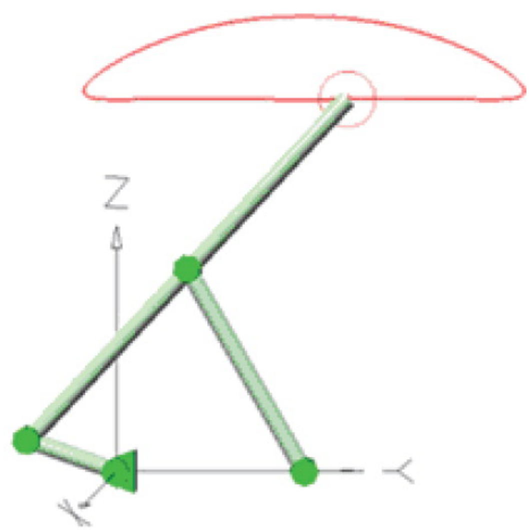


a)



b)

Figure 6. Coupler Curve Drawn by: a) Formula b) Dynamic analysis software

4. Generating Watt's six-bar obtained from optimal four-bar linkage

According to Roberts-Chebishev theorem three different planar four-bar linkages will trace identical coupler curve [10]. The procedure of drawing the two other linka; Privacy are called cognates of the four-bar is described in figure 7.

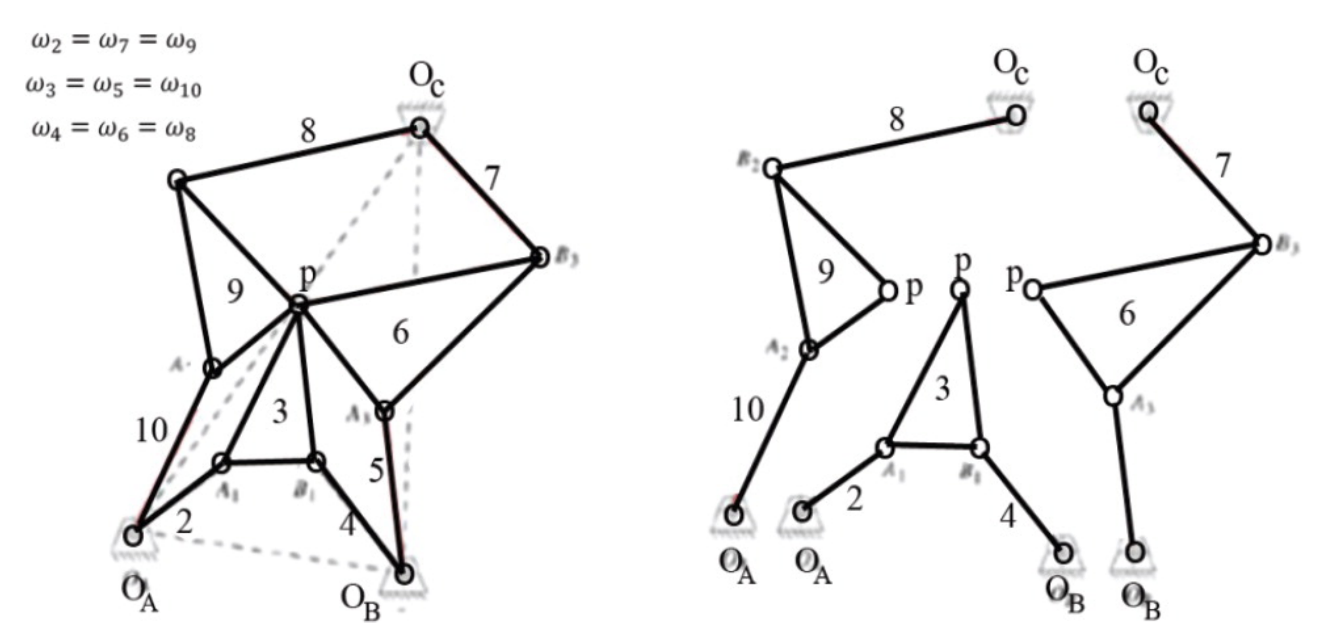


Figure 7. Procedure of drawing cognates of a mechanism

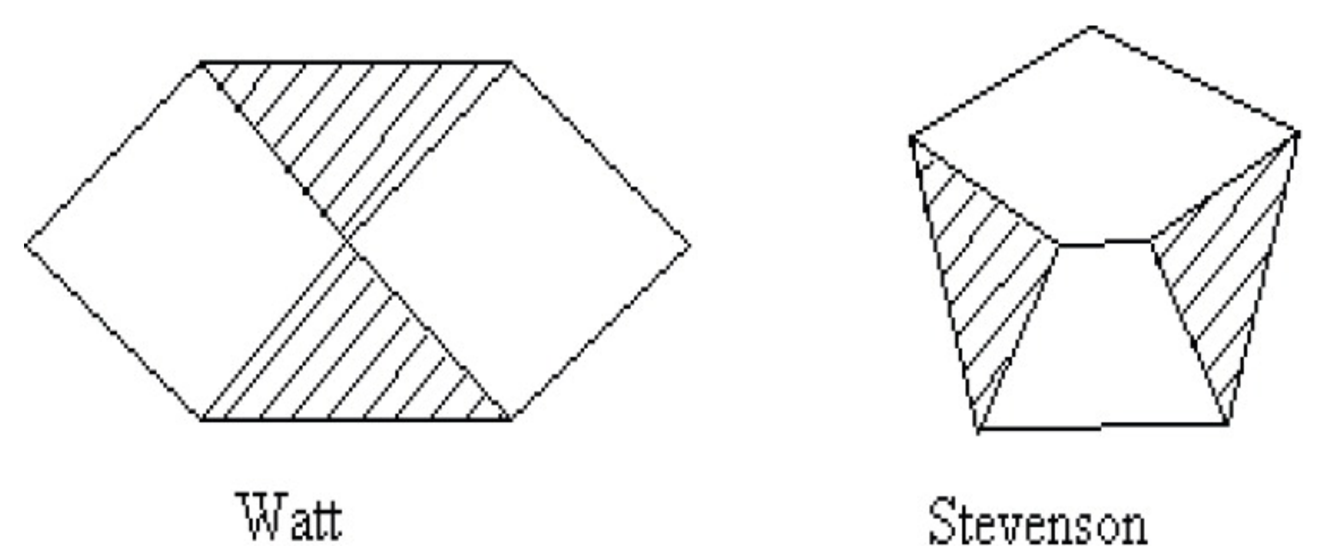


Figure 8. Two basic kinematic chains belonging to the six-bar 1 degree of freedom class of mechanisms

Figure 9 shows how we can obtain a Watt six-bar linkage by combining a four-bar linkage with it's cognate. Because of equality of angular velocity of rockers in both linkaɡ Privacy possible to couple them as Figure 9e. Figure 9f proves that combination of any four-bar linkage with one of it's cognates and elimination of to redundant links will result in a six-bar of Watt type.

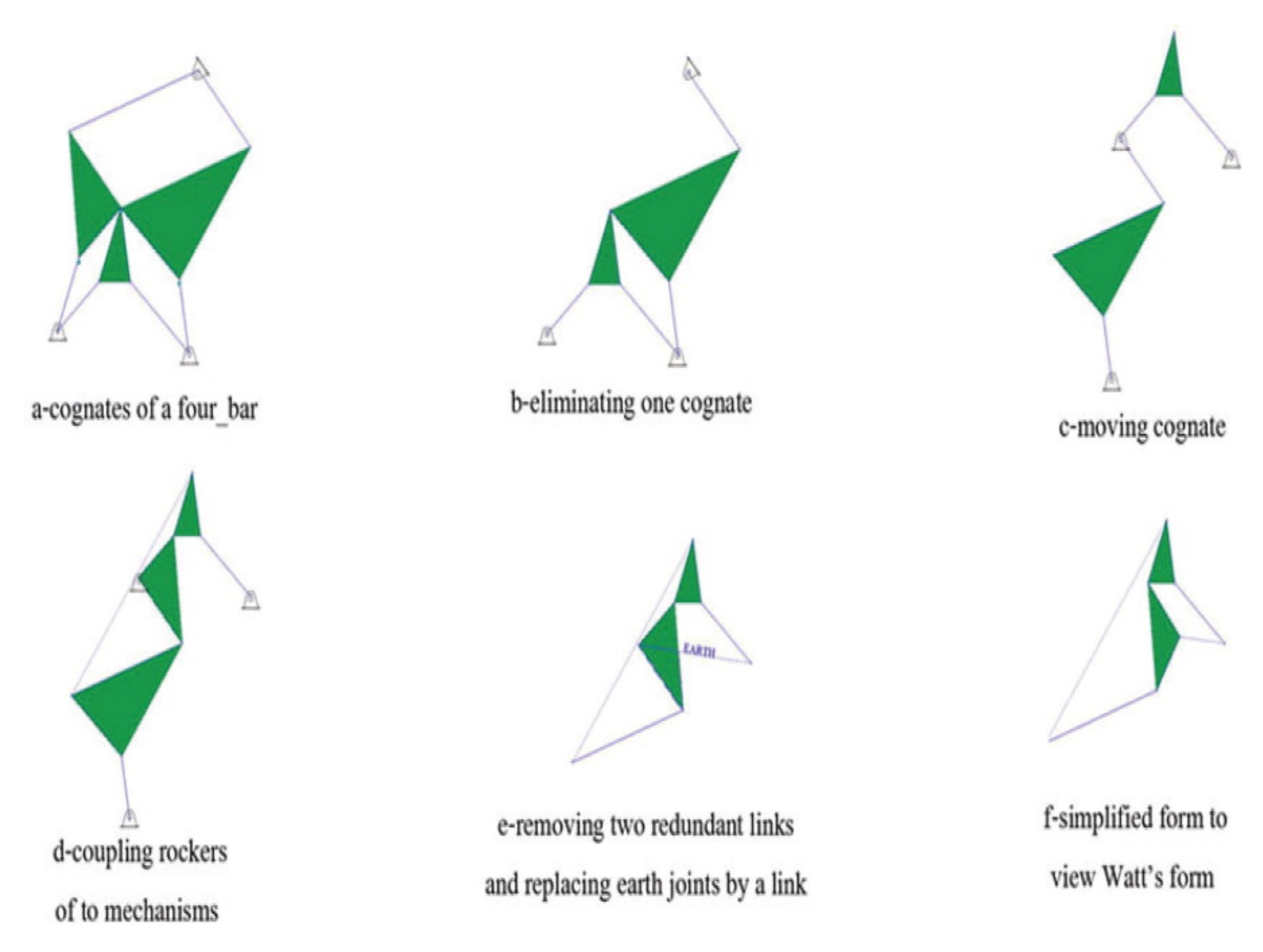


Figure 9. Procedure of obtaining Watt six-bar linkage from optimal four-bar

The same procedure can be applied to our optimal four-bar linkage as shown in Figure 10. Since two cognates generate exactly the same coupler curve, the link connecting them will always move parallel to it's initial position.

Inspite of it's appearance, the obtained mechanism is a Watt type with just one degree of freedom. Figure 11.a shows the position of the center of mass of a common type of six-legged hexapod robots. During a straight walk along x, y which represents the center of mass is oscillating constantly. This change in center of mass position is not desired and will result in waste of energy.

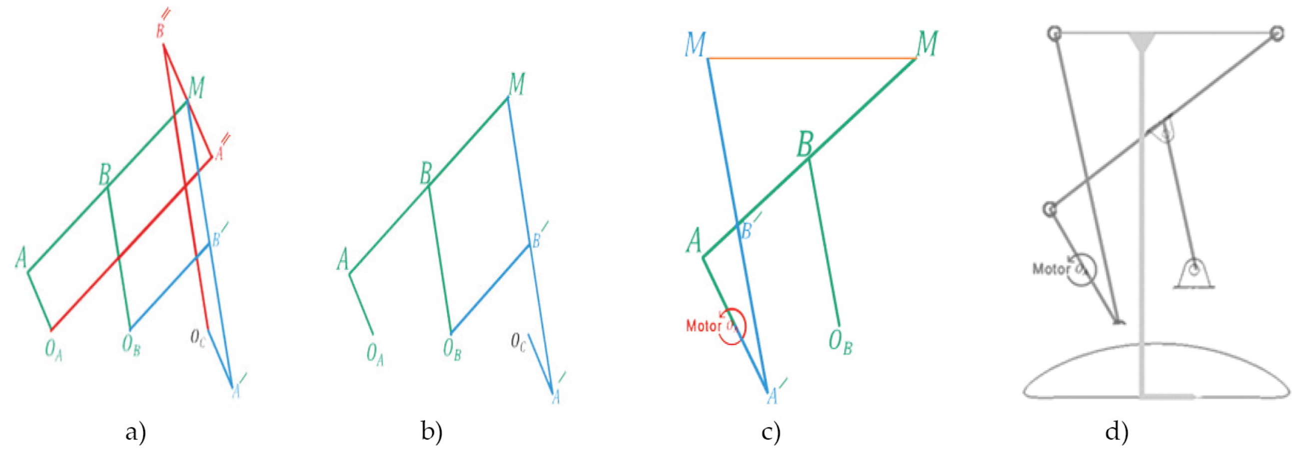
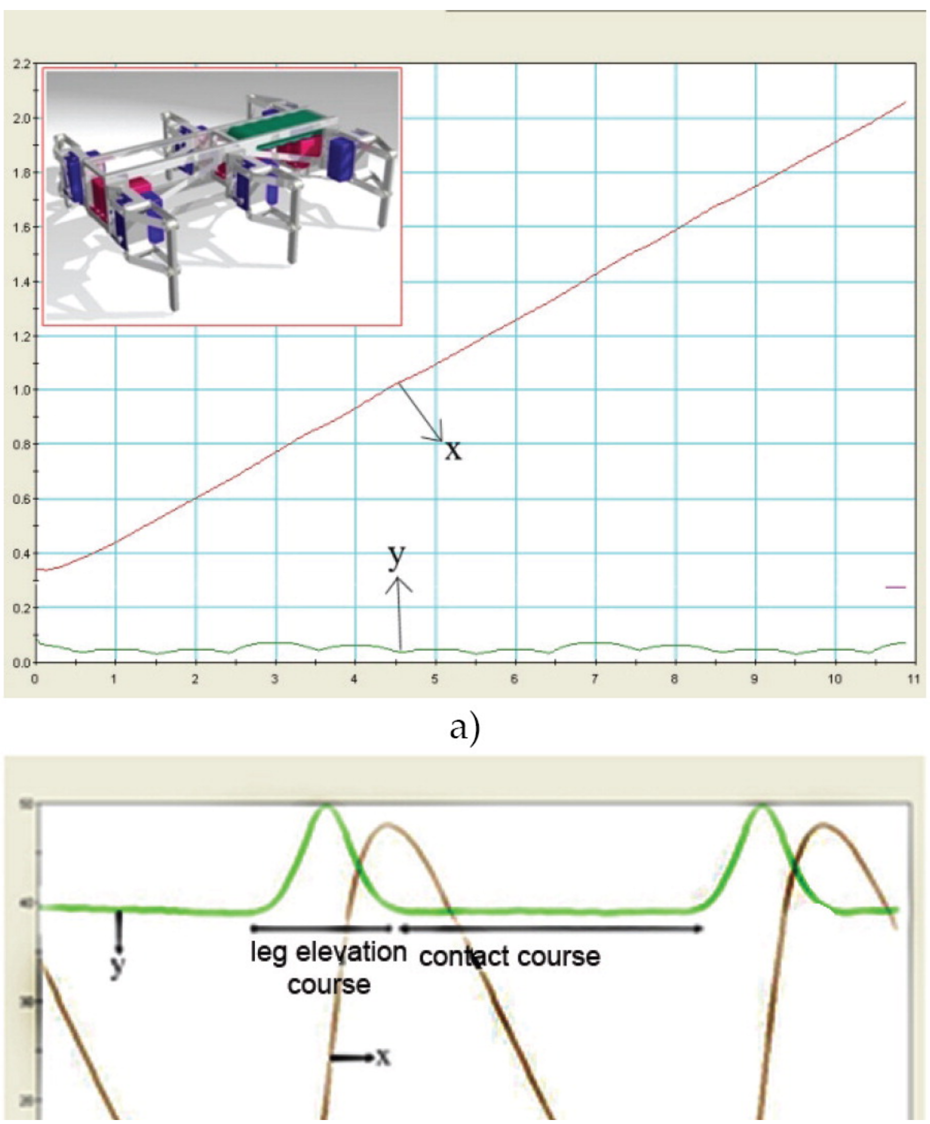
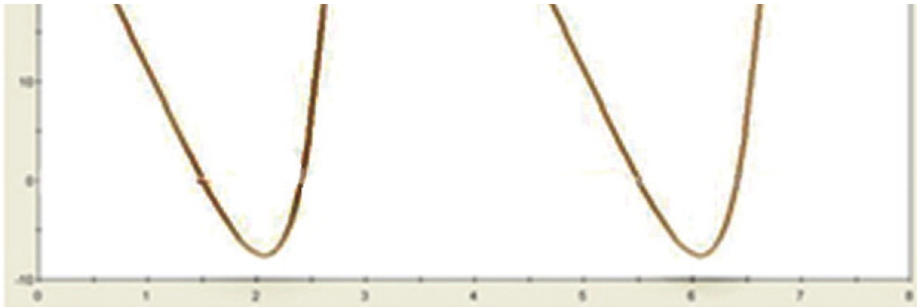


Figure 10. Different stages of establishing the Watt's six-bar linkage with leg connected to straight and parallel moving link





b)

Figure 11. a) Center of mass position of a hexapod versus time; b) Position of each point on the leg versus time [11]

Figure 11.b shows position of each point in a leg of the proposed six-bar linkage. Compared with figure 11.a, the leg moves completely straight in the contact course with the terrain. It means there is no change in center of mass position in vertical direction during the movement.

5. Conclusion

This paper describes the procedure of optimal synthesis of a four-bar linkage, with the application of genetic algorithm and combination of cognates of the optimal mechanism to achieve a Watt's six-bar linkage. Because the path generated by this mechanism is almost straight in one section and completely parallel, this six-bar is very suitable for legged robots. The coupler curve is symmetric and has no crunodes.

Parallel motion of the legs allows wide contact area of the leg with the terrain, which results in decrease of the number of the legs needed to hold the machine stable and also has less damaging effects to the terrain. The number of degrees of freedom is reduced to one, so the control system and dynamics of the motion will be much easier to analyze.

Unlike other straight line path generating mechanisms introduced above, this mechanism is a crank-rocker mechanism (i.e. there is no need to detach the linkage to achieve a complete coupler curve) and has no dead point. However in this paper the mechanism is used for legged robots, there would be other applications too.

The method is illustrated on the example of rectilinear motion of the coupler point, although the method can also be very efficiently applied when the path of the p